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EFFECT OF A PERTURBED WAVEFRONT ON THE POWER PATTERN OF AN ARRA--ETC(U)
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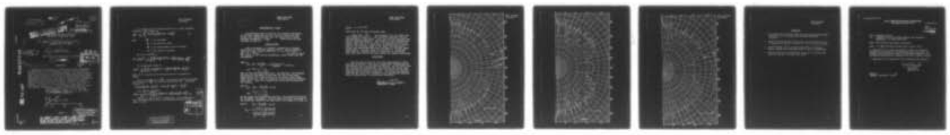

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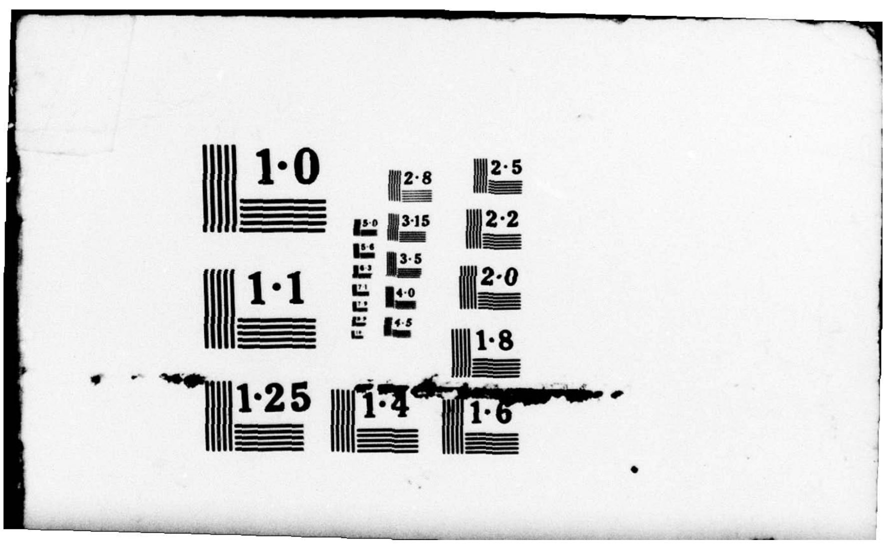
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NAVY UNDERWATER SOUND LABORATORY
NEW LONDON, CONNECTICUT 06320

(6) EFFECT OF A PERTURBED WAVEFRONT ON THE POWER PATTERN OF AN ARRAY.

by

(10) Benjamin F. Cron

NUSL Technical Memorandum No. 2211-99-70

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INTRODUCTION

A horizontal, geometrically spaced array, is under consideration for implantment in the Bermuda area. The problem is to obtain the pattern of the array for a perturbed wavefront. The first solution to this problem was obtained by Stocklin[1]. Stocklin considered a random phase at each receiver, that was Gaussian distributed and independent from receiver to receiver. This model was later extended by Berman and Berman[2] who considered correlated phases. Experimental values of phase (time delay) spreads and correlation of phases was obtained by Worley[3]. The purpose of this memorandum is to apply some of the experimental values to the theoretical equation and to find the effects on a possible sub-array configuration of the proposed array.

MATHEMATICAL EQUATIONS

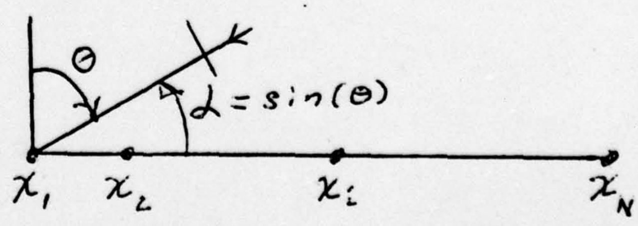

Figure 1

For a linear array, the complex pressure for a planar perturbed wavefront is (See Figure 1),

$$p = \frac{1}{N}\sum_{i=1}^{N} \exp\left[j\left(k\alpha x_i + \phi_i\right)\right] \tag{1}$$

where
- k is the wavenumber
- α is the direction cosine
- ϕ_i is the random phase at the ith receiver
- N is the number of receivers.

The power output P is

$$P = p\,p^* = \frac{1}{N^2}\sum_{j=1}^{N}\sum_{i=1}^{N} \exp\left[jk\alpha(x_i - x_j)\right]\exp\left[j(\phi_i - \phi_j)\right] \tag{2}$$

The average power output is

$$\langle P\rangle = \frac{1}{N^2}\sum_{j=1}^{N}\sum_{i=1}^{N} \exp\left[jk\alpha(x_i - x_j)\right]\left\langle \exp\left[j(\phi_i - \phi_j)\right]\right\rangle$$

where $\langle\ \rangle$, represents the expected value over an ensemble of signals.

Let us now assume that the ϕ's are Gaussian distributed with zero mean and variance σ_ϕ^2 and correlated spatially by ρ. Then (see Van Trees[4])

$$\left\langle \exp\left[j(\phi_i - \phi_j)\right]\right\rangle = \exp\left[-\sigma_\phi^2(1 - \rho_{ij})\right]$$

Using symmetry properties,

$$\langle P\rangle = \frac{1}{N^2}\left[N + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\cos\left[k\alpha(x_i - x_j)\right]\exp\left[-\sigma_\phi^2(1 - \rho_{ij})\right]\right] \tag{3}$$

Equation (3) will be used for computation. It is normalized so that a perfectly coherent wave in the broadside direction will be unity, i.e. 0db.

EXPERIMENTAL VALUES

Worley[3] found that at 400 Hz, the standard deviation of the path length was 1.5 ft. He also found that the phases were correlated and that the correlation versus distance resembles a $\frac{\sin x}{x}$ curve with the zero crossing at 750 ft.

COMPUTATIONS

Let us consider a 9 element, geometrically spaced array with the smallest spacing of 8 ft. and a geometric ratio of 1.551. Let $F = 400\ Hz$, $C = 5000\ ft./sec.$ and $\sigma_d = 1.5\ ft.$ In order to show the effect of the various parameters, three cases will be considered.

Case A

Let $\sigma_\phi = \frac{2\pi F \sigma_d}{C} = \frac{2\pi(400)1.5}{5000}$ radians

Let $\rho_{ij} = 1,\ i = j$
$\quad = 0,\ i \neq j$

This is the non-correlated case. In addition, the standard deviation of the phase is the same for all directions. Using these values and equation 3, we obtain the curves shown in Fig. 2. This is compared to the perfectly coherent plane wave. There is a significant difference between the two curves.

Case B

Let $\sigma_\phi = \frac{2\pi F \sigma_d}{C} \cos\theta$

Let $\rho_{ij} = 1,\ i = j$
$\quad = 0,\ i \neq j$

As the plane wave approaches end-fire, the standard deviation of the phase decreases. This is more realistic than case A. The perfectly coherent case and case B are compared in Fig. 3.

Case C $\sigma_\phi = \frac{2\pi F \sigma_d}{C} \cos\theta$

$$\rho_{ij} = \frac{\sin\left(\frac{\pi(x_i - x_j)\cos\theta}{L}\right)}{\left(\frac{\pi(x_i - x_j)\cos\theta}{L}\right)}$$

where $L = 750$ ft.

This case is the most realistic case.

Using this ρ_{ij} and equation (3), we obtain the curve shown in Figure 4. The perfectly coherent case is superimposed on this curve. Note that the two patterns are almost indistinguishable. The reason is that although the phases vary from trial to trial, for any given trial, although there may be a large phase difference due to travel time to the receiver, some of the other receivers have almost the same phase difference because of the high phase correlation. The sub array is about 472 ft. There are many pairs of elements that are separated by distances much less than 750 ft and these pairs are highly correlated.

CONCLUSION

For the case of 400 Hz, and from experimental values obtained by Worley, there is no appreciable degradation in the pattern due to a perturbed wavefront. We may infer that this is true for frequencies less than 400 Hz, since then, the standard deviation of the phase decreases and the correlation between phases increases. For higher frequencies than 400 Hz, there will probably be an appreciable degradation of pattern due to a perturbed wavefront.

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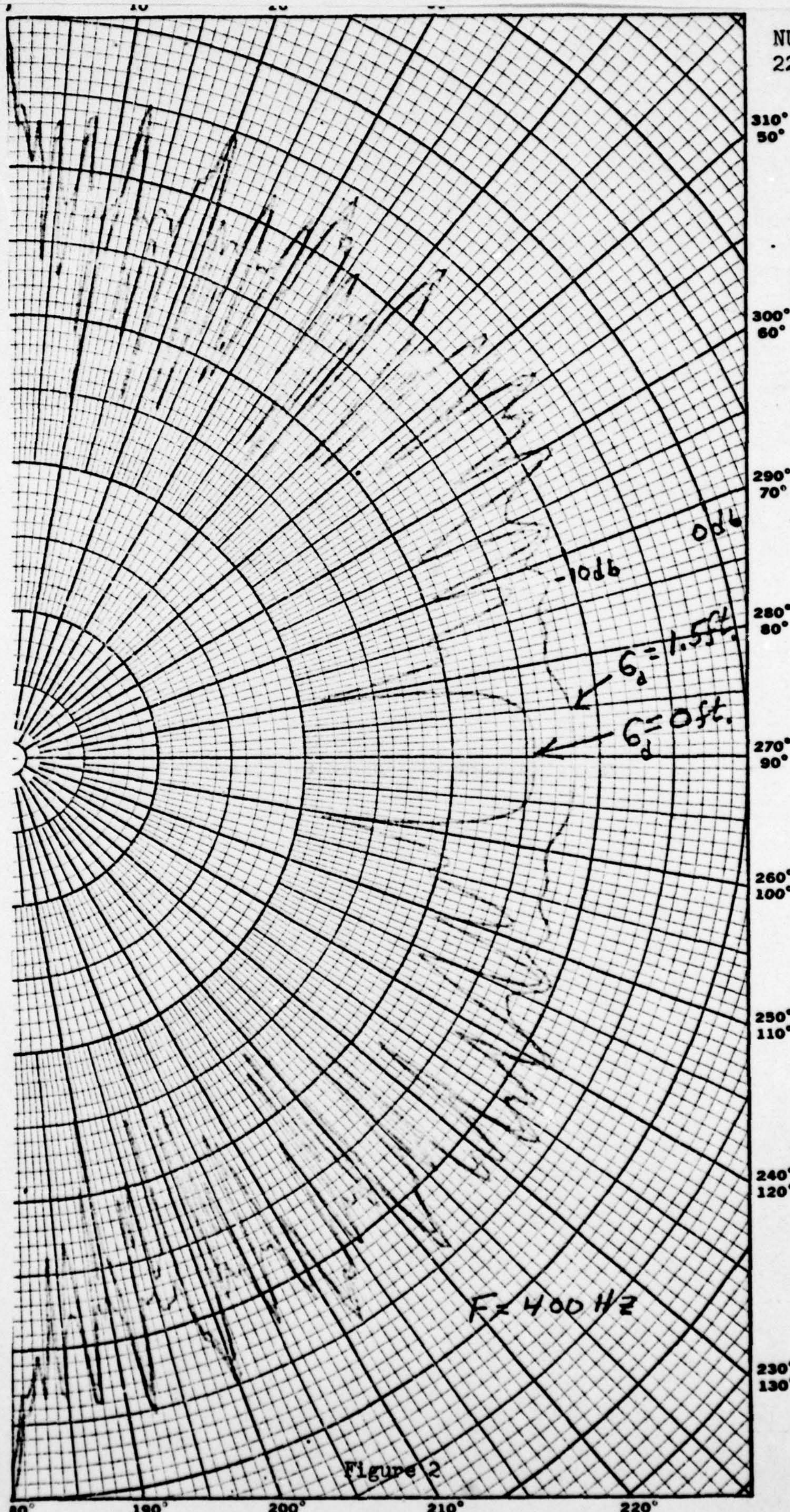
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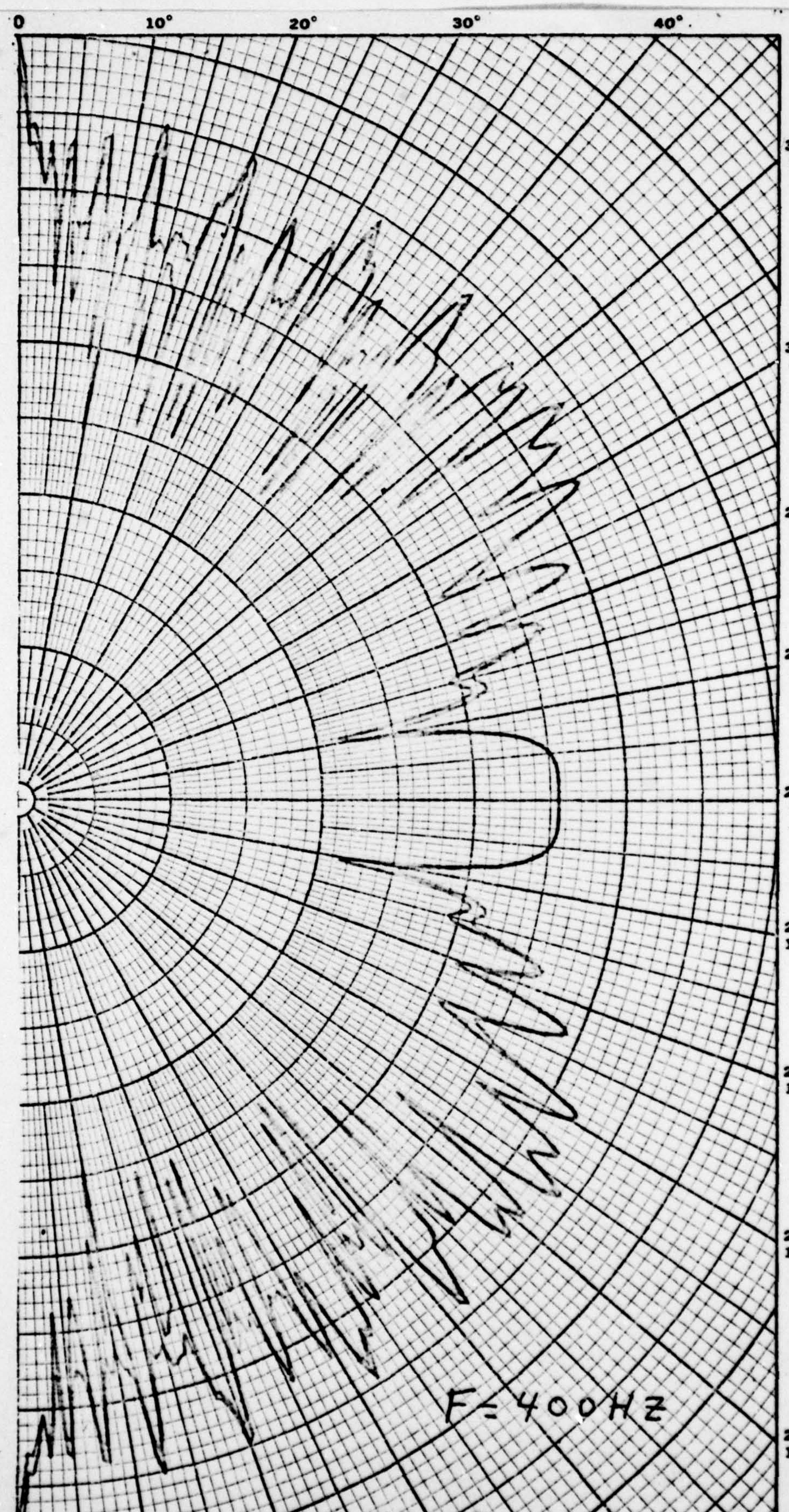
60°
290°
70°
280°
80°
270°
90°
260°
100°
250°
110°
240°
120°
230°
130°
0db
-10db
6_d=1.5ft.
6_d=0ft.
F=400 HZ
180°
190°
200°
210°
220°

Figure 2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250°
110°
240°
120°
230°
130°
F = 400HZ
90°
190°
200°
210°
220°
Figure 3

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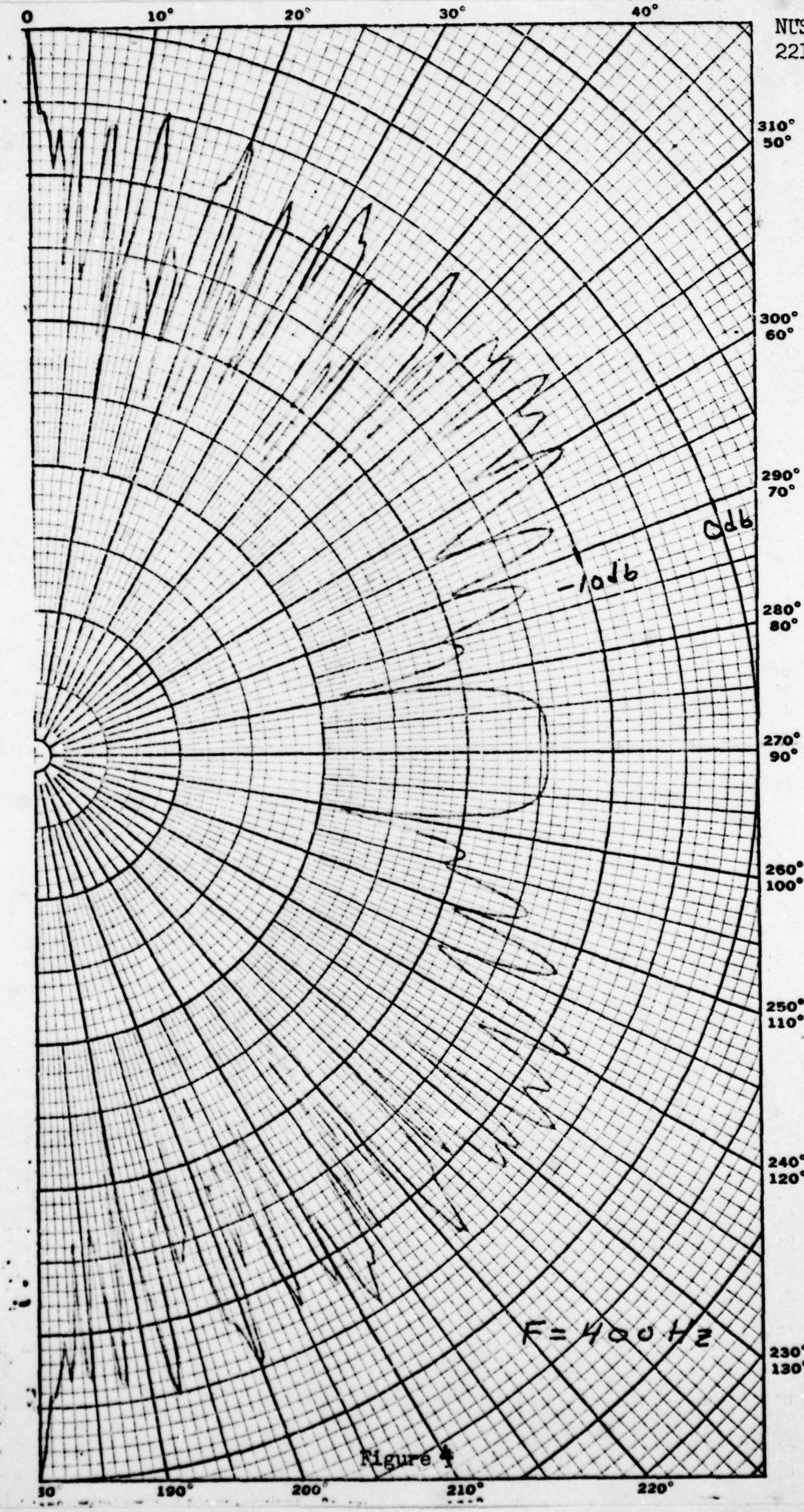

Figure 4

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3. Artemis Handbook, Vol II, Acoustic Propagation, R.D. Worley, Editor, Columbia University, Hudson Lab., 2 Feb 66 (Confidential)

4. Detection, Estimation and Modulation Theory by H VanTrees, John Wiley and Sons Inc., New York, 1967, Pg 96.

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Encl: (1) NUSL Tech Memo 2211-99-70 dtd 24 Apr 70 Cy 55

1. Enclosure (1) is a write-up showing the effect of a perturbed wavefront on the pattern of a geometrically spaced line array. It is shown that perturbations, based on experimental evidence, do not deteriorate the pattern for frequencies less than or equal to 400 Hz.

2. Enclosure (1) is forwarded for your information and retention.

F. H. Hunt

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